### **Bubbles and Credit Constraints**

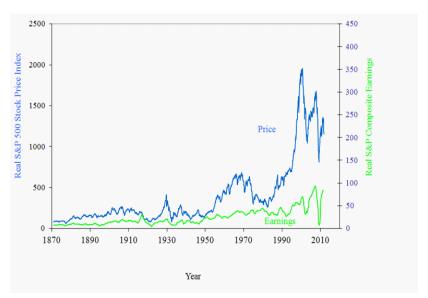
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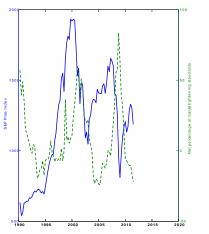
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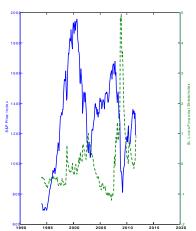
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### Motivation: US data



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#### Motivation: International evidence

- Rapid increases in stock prices are linked to
  - heavy capital inflows
  - rapid credit growth
  - rapid investment growth
- Collapse in stock prices were accompanied by
  - capital outflow
  - decreased investment
  - tightened credit
  - recession
- Collyns and Senhadji (2002), Gan (2007a,b), Goyal and Yamada (2004), Chaney, Sraer, and Thesmar (2009)

#### Motivation

- Bubbles and crashes are linked to booms and busts in credit market
- Is there any connection between rational bubbles/crashes and credit constraints?
- Provide a theory of credit-driven stock price bubble

## Major Historical Examples

- Tulipmania (1634-1638)
- The Mississippi Bubble (1719-1720)
- The South Sea Bubble (1720)
- The Bull Market of the Roaring Twenties (1924-1929)
- The Japanese "Bubble Economy" (1984-1989)
- dot-com bubble (1990s)
- China stock and property bubble (2007)
- US housing market bubbles

### Types of Bubbles

- Credit-driven stock price bubbles
  - Bubbles are accompanied with credit booms
  - Bubbles are on productive assets
  - Bubbles occur in a sector or an industry (Miao and Wang (2011a,b))
  - Recurrent bubbles with firm entry (Miao and Wang (2011c))
- Bubbles in prices of land, gold, paintings, money, etc.
- Rational vs Irrational bubbles (DeLong et al (1990), Scheinkman and Xiong (2003), Xiong and Yu (2011))

#### **Basic Intuition**

Asset pricing equation

$$P_t = D_t + \frac{P_{t+1}}{1+r}$$

- Exogenous payoffs  $D_t$  and discount rate r
- Solving forward

$$P_t = \underbrace{\sum_{s=0}^{\infty} \frac{D_{t+s}}{\left(1+r\right)^s}}_{ ext{fundamental}} + \underbrace{B_t}_{ ext{bubble}}, \quad B_t = \frac{B_{t+1}}{1+r}$$

Transversality condition rules out bubbles

### Key Issues

- Rational bubbles are fragile (Santos and Woodford (1997))
- A necessary condition is  $g \ge r$  or PV of consumption is infinity (Dynamically inefficient OLG)
- Existing theories typically study bubbles on assets with zero payoff or exogenously given payoffs
- What about bubbles on reproducible or productive assets? Dividends are endogenous!
- Do bubbles crowd out or crowd in capital?
- Welfare and policy implications of bubbles?

## Our Story

- Firms face stochastic investment opportunities, and borrow to finance investment subject to endogenous credit constraints
- Pledge firm assets (capital) as collateral (borrow against firm value)
- Collateral value may contain bubbles
- Positive feedback loop Optimistic beliefs about asset values (bubbles)
   ⇒ raise collateral value ⇒ raise lending against these assets ⇒
   raise investment ⇒ raise firm value or asset value ⇒ justify initial beliefs
- Bubbles are self-fulfilling
- Another equilibrium without bubble
- Stochastic bubbles, burst of bubbles ⇒ recession (No shocks to fundamentals)
- Bubbles on both intrinsically useless assets and productive assets can coexist

#### A Basic Model

- Discrete time t = 0, dt, 2dt, ... Continuous time  $dt \rightarrow 0$
- No aggregate uncertainty
- Risk neutral (can be relaxed) households supply labor inelastically (one unit).

$$\sum_{t\in\{0,dt,2dt,\ldots\}}e^{-rt}C_tdt,$$

- Can introduce endogenous labor supply
- A continuum of firms indexed by  $j \in [0, 1]$ , with technology:

$$Y_t^j = (K_t^j)^{lpha} (N_t^j)^{1-lpha}, \quad lpha \in (0,1)$$
 .

- Can introduce capacity utilization
- Solve static labor choice:

$$\max_{N_t^j} F\left(K_t^j, N_t^j\right) - w_t N_t^j = R_t K_t^j,$$



### Heterogeneity

 Investment opportunities arrive independently across firms and over time

$$\mathcal{K}_{t+dt}^{j} = \left\{ egin{array}{ll} \left(1-\delta dt
ight)\mathcal{K}_{t}^{j} + \mathit{I}_{t}^{j} & ext{with probability }\pi dt \\ \left(1-\delta dt
ight)\mathcal{K}_{t}^{j} & ext{with probability }1-\pi dt \end{array} 
ight.,$$

Can use idiosyncratic investment specific shocks with continuous distribution

$$K_{t+dt}^{j} = (1 - \delta dt) K_{t}^{j} + \varepsilon_{t}^{j} I_{t}^{j} dt$$

Can use idiosyncratic productivity shocks

$$Y_t^j = (A_t^j K_t^j)^{\alpha} (N_t^j)^{1-\alpha}$$



### Credit Constraints

ullet Intra-period loans (can be relaxed)  $L_t^j$ ,

$$I_t^j \le R_t K_t^j + L_t^j$$

- Assume no equity financing (can be relaxed)
- Let  $V_t(X^j)$  denote the market value of assets  $X^j$
- Credit constraint:

$$L_t^j \leq e^{-rdt} V_{t+dt}(\xi K_t^j).$$

- Pledge assets  $\xi K_t^j$  as collateral (effectively firm value)
- $oldsymbol{\delta}$  represents financial frictions
- Kiyotaki and Moore (1997) collateral constraint

$$L_t^j \leq \xi Q_t K_t^j$$
.



### Optimal Contract with Limited Commitment

- Albuquerque and Hopenhayn (2004), Alvarez and Jermann (2000), Jerman and Quadrini (2010)
- Borrow  $L_t^j$  and repay  $L_t^j$  only when investment opportunity arrives
- If firm defaults, lender seizes assets  $\xi K_t^j$
- Assets are not specific to the owner
- Lender reorganizes the firm and obtains going-concern value  $e^{-rdt}V_{t+dt}(\xi K_t^j)$
- The firm has all the bargainning power and the lender gets the threat value  $e^{-rdt}V_{t+dt}(\xi K_t^j)$
- Incentive constraint:

$$\underbrace{e^{-rdt} V_{t+dt}((1-\delta dt) \, K_t^j + I_t^j) - L_t^j}_{\text{Not default}} \\ \geq \underbrace{e^{-rdt} V_{t+dt}((1-\delta dt) \, K_t^j + I_t^j) - e^{-rdt} V_{t+dt}(\xi K_t^j)}_{\text{Default}}$$

# Firm's Problem (Optimal Contract)

Bellman equation

$$\begin{array}{ll} V_t(\mathit{K}_t^j) & = & \max_{\mathit{I}_t^j,\mathit{L}_t^j} \; R_t \mathit{K}_t^j dt - \pi \mathit{I}_t^j dt + \mathrm{e}^{-rdt} V_{t+dt} ((1-\delta dt) \, \mathit{K}_t^j + \mathit{I}_t^j) \pi dt \\ & + \mathrm{e}^{-rdt} V_{t+dt} ((1-\delta dt) \, \mathit{K}_t^j) (1-\pi dt), \end{array}$$

subject to

$$I_t^j \le R_t K_t^j + L_t^j$$

$$L_t^j \le e^{-rdt} V_{t+dt}(\xi K_t^j)$$

Not a contraction mapping!

# Competitive Equilibrium

- Aggregation  $K_t=\int_0^1 K_t^j dj$ ,  $I_t=\int_0^1 I_t^j dj$ ,  $N_t=\int_0^1 N_t^j dj$ , and  $Y_t=\int_0^1 Y_t^j dj$
- Households and firms optimize and markets clear

$$egin{array}{lcl} N_t &=& 1, \ C_t + \pi I_t &=& Y_t, \ K_{t+dt} &=& \left(1 - \delta dt
ight) K_t + I_t \pi dt. \end{array}$$

### **Usual Solution**

Firm value takes the form (Hayashi (1982)):

$$V_t(K_t^j) = v_t K_t^j$$

Verify:

$$\begin{aligned} v_t K_t^j &= \max_{l_t^j} R_t K_t^j dt - \pi I_t^j dt + \underbrace{e^{-rdt} v_{t+dt}}_{Q_t} \pi I_t^j dt \\ &+ \underbrace{e^{-rdt} v_{t+dt}}_{Q_t} \left(1 - \delta dt\right) K_t^j, \\ &I_t^j \leq R_t K_t^j + \underbrace{e^{-rdt} v_{t+dt}}_{Q} \xi K_t^j \end{aligned}$$

ullet Need  $Q_t > 1$  for the investment and collateral constraint to bind

#### Bubble Solution

• Firm value takes the form:

$$V_t(K_t^j) = \underbrace{v_t K_t^j}_{\text{fundamental}} + \underbrace{b_t}_{\text{bubble}}, \ b_t > 0$$

Positive feedback loop:

$$\begin{aligned} v_t \mathcal{K}_t^j + b_t &= \max_{l_t^j} \ R_t \mathcal{K}_t^j dt - \pi I_t^j dt + \underbrace{e^{-rdt} v_{t+dt}}_{Q_t} \pi I_t^j dt \\ &+ \underbrace{e^{-rdt} v_{t+dt}}_{Q_t} \left(1 - \delta dt\right) \mathcal{K}_t^j + \underbrace{e^{-rdt} b_{t+dt}}_{B_t}, \\ I_t^j &\leq R_t \mathcal{K}_t^j + \underbrace{e^{-rdt} v_{t+dt}}_{Q_t} \xi \mathcal{K}_t^j + \underbrace{e^{-rdt} b_{t+dt}}_{B_t}, \end{aligned}$$

• Need  $Q_t > 1$  for the investment and collateral constraint to bind

### Continuous Time Equilibrium System

Suppose  $Q_t > 1$ . Then  $(B_t, Q_t, K_t)$  satisfy:

$$\dot{\mathcal{B}}_t = r\mathcal{B}_t - \mathcal{B}_t\pi(Q_t-1),$$
  $\dot{\mathcal{Q}}_t = (r+\delta)\,Q_t - \mathcal{R}_t - \pi(\mathcal{R}_t+\xi Q_t)(Q_t-1),$   $\dot{\mathcal{K}}_t = -\delta\mathcal{K}_t + \pi(\mathcal{R}_t\mathcal{K}_t+\xi Q_t\mathcal{K}_t+\mathcal{B}_t),~\mathcal{K}_0$  given,

and the transversality condition:

$$\lim_{T\to\infty} e^{-rT} Q_T K_T = 0, \quad \lim_{T\to\infty} e^{-rT} B_T = 0,$$

- Bubbleless equilibrium:  $B_t = 0$
- Bubbly equilibrium:  $B_t \neq 0$

# Why bubbles?

- Santos and Woodford condition? r > 0, zero economic growth
- Bubble dynamics

$$\underbrace{rac{\dot{B}_t}{B_t}}_{ ext{capital gains}} + \underbrace{\pi(Q_t-1)}_{ ext{dividend yields}} = r$$

- Bubble is productive!
- TV cannot rule out bubble
- Bubbleless SS is dynamically efficient because  $K^* < K_{GR} = (\delta/\alpha)^{\frac{1}{\alpha-1}}$   $\Longrightarrow$  Tirole (1985) condition does not apply

### Bubbly Equilibrium: Steady State

There exists  $(B, Q_b, K_b)$  satisfying

$$\frac{B}{K_b} = \frac{\delta}{\pi} - \frac{r+\delta+\xi}{1+r} \frac{r+\pi}{\pi} > 0,$$

$$Q_b = \frac{r}{\pi} + 1 > 1,$$

$$\alpha \left(K_b\right)^{\alpha-1} = \frac{(1-\xi)r+\delta}{1+r} \left(\frac{r}{\pi} + 1\right),$$

if and only if

$$0 < \xi < \frac{\delta (1-\pi)}{r+\pi} - r. \tag{1}$$

In addition, (i)  $Q_b < Q^*$ , (ii)  $K_{GR} > K_E > K_b > K^*$ , and (iii) the bubble-asset ratio  $B/K_b$  decreases with  $\xi$ .



## **Bubbly Equilibrium Dynamics**

Suppose condition (1) holds. Then both the bubbly steady state  $(B, Q_b, K_b)$  and the bubbleless steady state  $(0, Q^*, K^*)$  are local saddle points for the nonlinear system for  $(B_t, Q_t, K_t)$ .

• Different from indeterminancy (Benhabib or Farmer)

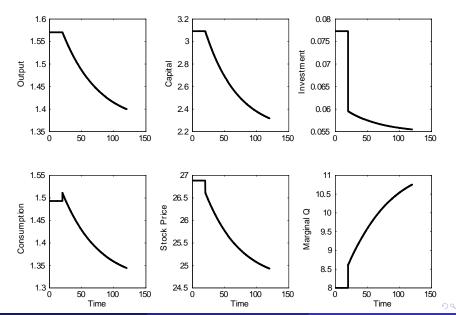
#### Robustness

- Intertemporal Borrowing and Saving
- Idiosyncratic Investment-Specific Shocks: bubbles can arise even if capital can be fully pledgeable, e.g,  $\xi=1$
- Idiosyncratic Productivity Shocks: bubble can generate endogenous TFP through capital reallocation

#### Stochastic Bubbles

- Blanchard and Watson (1982), Weil (1987)
- Suppose a bubble exists initially,  $B_0 > 0$ .
- Between t and t+dt, there is probability  $\theta dt$  that the bubble bursts,  $B_{t+dt}=0$ . Once it bursts, it will never be valued again in the future so that  $B_{\tau}=0$  for all  $\tau \geq t+dt$ .
- With probability  $1 \theta dt$ , the bubble persists so that  $B_{t+dt} > 0$ .
- Take the continuous time limits as  $dt \rightarrow 0$ .

# Stochastic Bubble Equilibrium Dynamics



### Capacity utilization

Production function

$$Y_t^j = (u_t^j K_t^j)^{\alpha} \left( N_t^j \right)^{1-\alpha}, \tag{2}$$

where  $u_t^j$  represents the capacity utilization rate.

• Deprecation rate

$$\delta_t^j = \varphi(u_t^j),\tag{3}$$

where  $\phi$  is an increasing and convex function.

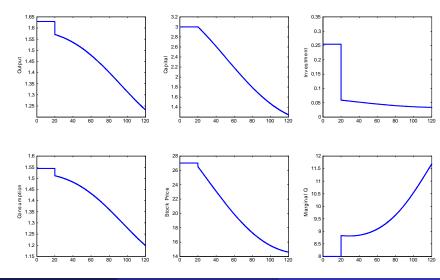
At equilibrium

$$u_t^j = u(Q_t), (4)$$

where  $u'(Q_t) < 0$ 



# Capacity utilization



# Public Assets and Credit Policy

- Monetary policy: Lean vs clean debate
- What types of bubbles? (Mishkin) What causes bubbles?
- Inefficiency comes from credit constraints
- Bubbles help relax these constraints, while the collapse of bubbles tightens them
- Government can supply liquidity to the firms by issuing public bonds.
   These bonds are backed by lump-sum taxes.

$$M_t P_t = T_t dt + M_{t+dt} P_t$$
,

 Bubble on unbacked public bonds can exist with household short sales constraint.

# Optimal Credit Policy

• Suppose assumption (1) holds. Let the government issues a constant value  $D = P_t M_t$  of government debt given by

$$D_t = D \equiv K_E \left( \delta \frac{1-\pi}{\pi} - r - \xi \right) > 0,$$

which is backed by lump-sum taxes  $T_t = T \equiv rD$  for all t. Then this credit policy will eliminate the bubble on firm assets and make the economy achieve the efficient allocation.

 Unbacked public assets (intrinsically useless assets) can have a bubble value when households face short sales constraint. This bubble may coexist with stock price bubbles and boost the economy when stock market bubbles burst. The equilibrium real allocation is identical to that with stock price bubbles only.

#### Conclusion

- We have provided an infinite-horizon model of a production economy with bubbles on productive assets
- Bubbles help relax collateral constraints and generate dividend yields
- Collapse of bubbles have adverse impact on the economy
- When public assets backed by lump-sum taxes are used as collateral, there exists a credit policy that can eleminate bubbles and make the economy achieve efficiency